The Prandtl Number Dependence of the Convective Heat Transfer Coefficient in a Turbulent Thermal Entrance Region

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The convective film heat transfer coefficient for turbulent flow in the thermal entrance regions of smooth closed channels can be predicted from several correlating equations appearing in the published literature (Abbrecht and Churchill, 1960; Aladyev, 1954; Knudsen and Katz, 1958). Basically these equations are empirical modifications to the conventional fully developed (far downstream) equation, usually written in the Nusselt number form

$$Nu = C_i \, Re^m \, Pr^n \tag{1}$$

The entrance length modifications include changes in the numerical values of m and C_i or in the use of an additional factor involving a function of L/D, the dimensionless distance from the start of the heat transfer section. The equations were based both on experiments in which the thermal entrance region was also a hydrodynamic entrance region (Aladyev, 1954; Knudsen and Katz, 1958) and where it was located in a fully developed turbulent flow channel (Abbrecht and Churchill, 1960).

The value of m, the Reynolds number exponent, was reported to be in the range 0.67 to 0.80, with the lowest value reported for the shortest L/D. The Prandtl number exponent used in the empirical equations was either 0.33 (Knudsen and Katz, 1958) or 0.4 (Abbrecht and Churchill, 1960; Aladyev, 1954), independent of L/D, values characteristic of the fully developed heat transfer equation. In these studies, the Prandtl number was not systematically varied, and the choice of exponent n was dictated more by the desire of the experimenter to maintain the form of the far downstream equation than by experimental evidence.

In this work the objective was to determine the best value of the Prandtl number exponent in the thermal entrance region for a particular flow geometry; a heated flat wall of $L/D = 1.25$ located in a smooth rectangular channel. Two sets of experiments were performed: one in which the hydrodynamic boundary layer was developed in an upstream section of $L/D = 20$ and a second where this boundary layer was disrupted by turbulence promoters located adjacent to the wall. The empirical values of C_i, m, and n for these cases were obtained by systematic variation of the flow velocity in the channel, the composition of the fluids (water and glycerol) and the bulk temperature of the fluid.

DESCRIPTION OF EXPERIMENT

The system consisted of a recirculating loop containing the test section assembly, a 0.21 m^3 open reservoir, a 0.75 kW circulating pump, and a calibrated flow meter. The test section was a stainless steel rectangular channel 5 cm $\times$ 1.3 cm in cross section and approximately 50 cm long. A 1.6-mm O.D. stainless steel clad Chromel/Alumel calibrated thermocouple measured the bulk fluid temperature. The heated surface consisted of one face of a 2.54 cm cube of copper imbedded in an epoxy matrix to provide thermal insulation. The face of the cube exposed to the fluid was machined flush with the epoxy insulation and installed in the channel flush with the adjacent wall surfaces.

An electric resistance heater on the back face of the cube provided the heat flux to the fluid. The AC heater was capable of supplying 25 watts. Three calibrated Chromel/Alumel thermocouples clad in a 0.51-mm O.D. stainless steel sheath were imbedded in the block at known locations (within 0.05 mm).

The entire test section was thermally insulated, and heating and cooling coils were provided in the reservoir such that bulk temperatures from 0 to 88°C could be maintained in the fluid entering the test section. Turbulence promoters were installed in the flow channel in one series of experiments. Three 0.16-mm diam. stainless steel wires were mounted transverse to the direction of flow as shown in Figure 1. The upstream and downstream wires touched the surface while the center wire was located 0.24 mm above the surface of the cube.

The experiments were conducted by setting the fluid bulk temperature and composition at various values and obtaining sets of temperature measurements (bulk temperature and block temperatures) as a function of fluid bulk velocity. Two heat meter power inputs were used, corresponding to heat fluxes of 38,738 watts/m^2-20,141 watts/m^2. The fluids tested were demineralized water and aqueous-glycerol solutions up to 30 wt. % of glycerol. The glycerol concentration was determined by specific gravity using a pycnometer, and the viscosity based on that concentration was checked with an Ostwald type viscometer, with agreement within 2 wt. %. Temperatures from 0° to 88°C were used, with the 0°C data obtained by maintaining an ice-water mixture in the reservoir.

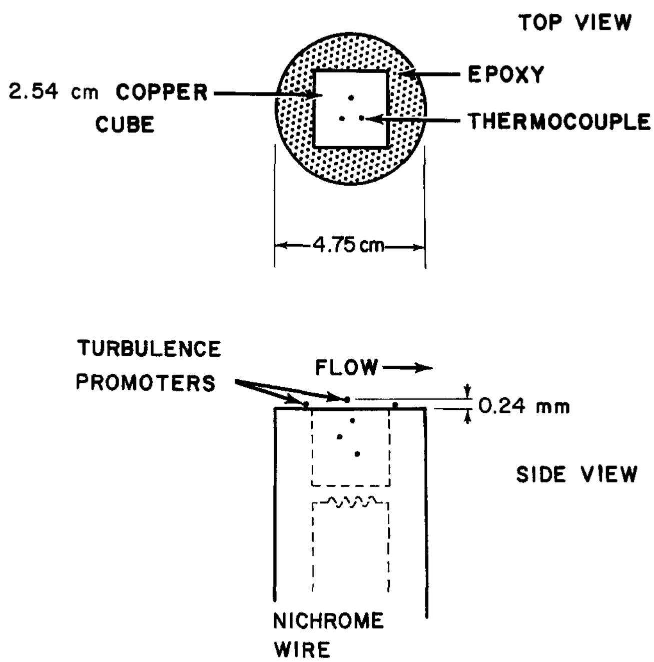

Fig. 1. Schematic of heat meter.

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Velocities ranged from 30 to 330 cm/s in the test section channel, corresponding to a Reynolds number (based on channel hydraulic diameter) from 10,000 to 200,000. The Prandtl number varied from 1.9 to 16.

DISCUSSION OF RESULTS

From measurements of power input to the heat meter and the temperatures within the copper block, the heat flux and surface temperature were calculated. The heat transfer coefficient was then determined from the wall and bulk fluid temperatures and the heat flux.

The exponent for the Reynolds number then was obtained from a linear regression of log Nusselt number versus log Reynolds number for each bulk temperature. The data for the smooth surface using water are shown in Figure 2. The Prandtl number variation is due to the temperature variation while the Reynolds number (at constant Pr) varied due to velocity. The mean slope from a combined least squares fit was 0.70. Similar plots for glycerol-water solutions and for the promoted surface yielded values in the range 0.70 to 0.72.

The Prandtl number dependence was determined by linear regression of log $Nu/Re^{0.7}$ against log Pr. Some of the data are shown plotted in Figure 3. The slopes obtained by statistical analysis were 0.35 for the smooth surface and 0.39 for the enhanced surface, regardless of whether water or glycerol-water solutions were used.

The resulting correlating equations for the data obtained in this study were

$$Nu = 0.122\ Re^{0.70}\ Pr^{0.35} \tag{2}$$

for the smooth surface, and

$$Nu = 0.136\ Re^{0.70}\ Pr^{0.39} \tag{3}$$

for the surface with detached turbulence promoters.

DISCUSSION

The present results substantially agree with the previous studies that the Reynolds number exponent decreases for heat transfer in thermal entrance regions. It also substantiates the inherent assumption that the Prandtl number exponent to use in smooth wall thermal entrance region heat transfer correlations is the same as for the fully developed thermal boundary layer case.

The results show that upsetting of the hydrodynamic boundary layer does not change the Reynolds number exponent but does increase the exponent of the Prandtl number. This increase accounts for about half of the total enhancement which resulted from the turbulence promoters at the lower temperatures (higher Prandtl numbers).

The results have several implications. First, they substantiate the validity of the existing correlations for predicting heat transfer in thermal entrance regions. Second, they shed light on the mechanism of heat transfer enhancement by boundary layer upset. Specifically, the results imply that when heat transfer is enhanced by periodic upsetting of the hydrodynamic boundary layer, the importance of molecular diffusion properties of the fluid is diminished [the Prandtl number exponent in (1) increases]. This result is in agreement with recently published results of convective film enhancement using periodic annular ridges (Webb et al., 1972).

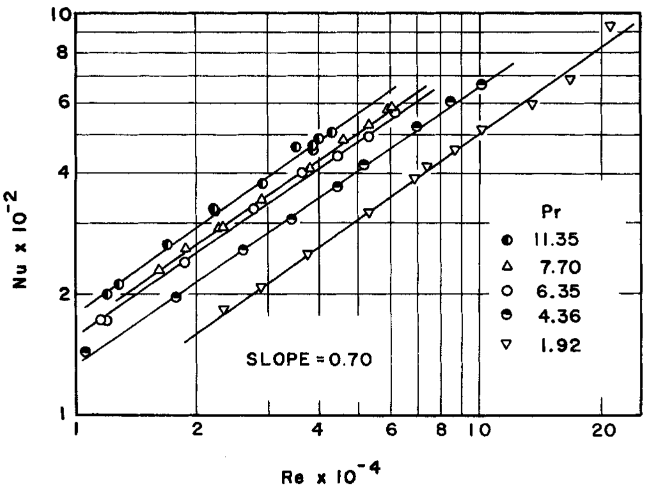

Fig. 2. Effect of Prandtl number and Reynolds number on the Nusselt number.

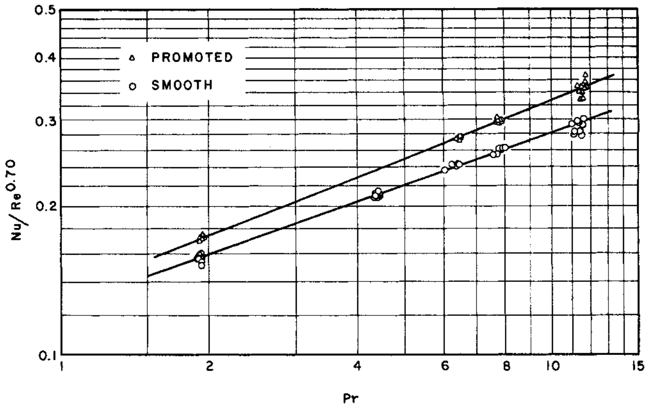

Fig. 3. Effect of Prandlt number on Nusselt number.

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NOTATION

- C_i = convective constant in Equation (1)
- D = hydraulic diameter of flow channel
- L = heated length of flow channel
- m = Reynolds number exponent in Equation (1)
- n = Prandtl number exponent in Equation (1)
- Nu = Nusselt number based on hydraulic diameter
- Pr = Prandtl number
- Re = Reynolds number based on hydraulic diameter

LITERATURE CITED

Abbrecht, P. H., and S. W. Churchill, "The Thermal Entrance Region in Fully Developed Turbulent Flow," *AIChE J.*, **6**, 268 (1960).

Aladyev, I. T., "Experimental Determination of Local and Mean Coefficients of Heat Transfer for Turbulent Flow in Pipes," NACA TM 1356 (1954).

Knudsen, J. G., and D. L. Katz, *Fluid Dynamics and Heat Transfer*, p. 401, McGraw-Hill, New York (1958).

Webb, R. L., E. R. G. Eckert, and R. J. Goldstein, "Generalized Heat Transfer and Friction Correlations for Tubes with Repeated Rib Roughness," *Intern. J. Heat Mass Transfer*, **15**, 180 (1972).

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